DAY THREE

Scalar and Vector

Learning & Revision for the Day

- Scalar and Vector Quantities
- Laws of Vector Addition
- Substraction of Vectors
- Multiplication or Division of a Vector by a Scalar
- Product of Vects
- Resolution of vector
- Relative velocity
- Motion in a Plane
- Projectile Motion

Scalar and Vector Quantities

A **scalar quantity** is one whose specification is completed with its magnitude only. e.g. mass, distance, speed, energy, etc.

A **vector quantity** is a quantity that has magnitude as well as direction. Not all physical quantities have a direction. e.g. velocity, displacement, force, etc.

Position and Displacement Vectors

A vector which gives position of an object with reference to the origin of a coordinate system is called position vector.

The vector which tells how much and in which direction on object has changed its position in a given interval of time is called displacement vector.

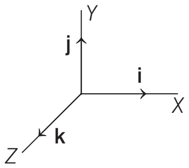
General Vectors and Notation

- **Zero Vector** The vector having zero magnitude is called **zero vector** or **null vector**. It is written as 0. The initial and final points of a zero vector overlap, so its direction is arbitrary (not known to us).
- **Unit Vector** A vector of unit magnitude is known as an **unit vector**. Unit vector for **A** is $\hat{\mathbf{A}}$ (read as A cap).

$$\mathbf{A} = \underset{\substack{|\\ \text{Magnitude}}}{A}\,\hat{\mathbf{A}} \text{ ——— Direction}$$

- **Orthogonal Unit Vectors** The unit vectors along X-axis,s, Y-axis and Z-axis are denoted by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. These are the orthogonal unit vectors.

$$\hat{\mathbf{i}} = \frac{\mathbf{x}}{x}, \hat{\mathbf{j}} = \frac{\mathbf{y}}{y}, \hat{\mathbf{k}} = \frac{\mathbf{z}}{z}$$



- **Parallel Vector** Two vectors are said to be parallel, if they have same direction but their magnitudes may or may not be equal.
- **Antiparallel Vector** Two vectors are said to be anti-parallel when
  - (i) both have opposite direction
  - (ii) one vectors is scalar non zero negative multiple of another vector.
- **Collinear Vector** Collinear vector are those which act along same line.
- **Coplanar Vector** Vector which lies on the same plane are called coplanar vector.
- **Equal Vectors** Two vectors **A** and **B** are equal, if they have the same magnitude and the same direction.

Laws of Vector Addition

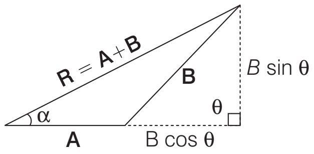
1. Triangle Law

If two non-zero vectors are represented by the two sides of a triangle taken in same order than the resultant is given by the closing side of triangle in opposite order, i.e.

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

The resultant R can be calculated as

$$|\mathbf{A} + \mathbf{B}| = R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

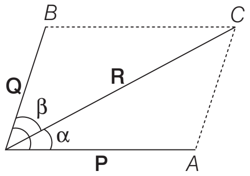


If resultant R makes an angle α with vector A, then

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

2. Parallelogram Law

According to parallelogram law of vector addition, if two vector acting on a particle are represented in magnitude and direction by two adjacent side of a parallelogram, then the diagonal of the parallelogram represents the magnitude and direction of the resultant of the two vector acting as the particle.



i.e. $\mathbf{R} = \mathbf{P} + \mathbf{Q}$

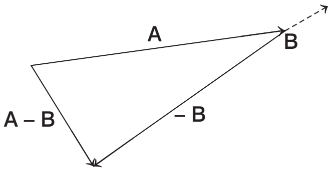
Magnitude of the resultant **R** is given by

$$|\mathbf{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \Rightarrow \tan\beta = \frac{P\sin\theta}{Q + P\cos\theta}$$

Subtraction of Vectors

Vector subtraction makes use of the definition of the negative of a vector. We define the operation $\mathbf{A} - \mathbf{B}$ as vector $-\mathbf{B}$ added to vector **A**. $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$



Thus, vector subtraction is really a special case of vector addition. The geometric construction for subtracting two vectors is shown in the above figure.

If θ be the angle between A and B,

then $|\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

If the vectors form a closed n sided polygon with all the sides in the same order, then the resultant is zero.

Multiplication or Division of a Vector by a Scalar

The multiplication or division of a vector by a scalar gives a vector. For example, if vector **A** is multiplied by the scalar number 3, the result, written as 3**A**, is a vector with a magnitude three times that of **A**, pointing in the same direction as **A**. If we multiply vector **A** by the scalar -3, the result is $-3\mathbf{A}$, a vector with a magnitude three times that of **A**, pointing in the direction opposite to **A** (because of the negative sign).

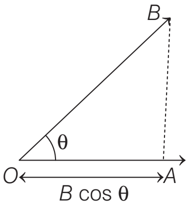
Product of Vectors

The two types of products of vectors are given below

Scalar or Dot Product

The scalar product of two vectors A and B is defined as the product of magnitudes of A and B multiplied by the cosine of smaller angle between them. i.e. $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

Properties of Dot Product



Component of vector B along A

- Dot product or scalar product of two vectors gives the scalar two vectors given the scalar quantity.
- It is commutative in nature.
  i.e. $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$.
- Dot product is distributive over the addition of vectors.
  i.e. $\mathbf{A}\cdot(\mathbf{B} + \mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$
- $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$, because angle between two equal vectors is zero.
- If two vectors A and B are perpendicular vectors, then
  $\mathbf{A}\cdot\mathbf{B} = AB\cos 90^\circ = 0$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$

The Vector Product

The vector product of **A** and **B**, written as $\mathbf{A}\times\mathbf{B}$, produces a third vector **C** whose magnitude is $\mathbf{C} = AB\sin\theta$. where, θ is the smaller of the two angles between **A** and **B**.

Because of the notation, $\mathbf{A}\times\mathbf{B}$ is also known as the **cross product**, and it is spelled as '**A** cross **B**'.

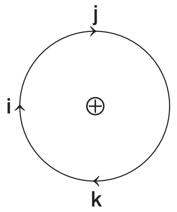
Properties of Cross Product

- Vector or cross product of two vectors gives the vector quantity.
- Cross product of two vectors does not obey the commutative law. i.e. $\mathbf{A}\times\mathbf{B}\neq\mathbf{B}\times\mathbf{A}$;

  Here, $\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$
- Cross product of two vectors is distributive over the addition of vectors.
$$\mathbf{A}\times(\mathbf{B}+\mathbf{C})=\mathbf{A}\times\mathbf{B}+\mathbf{A}\times\mathbf{C}$$
- Cross product of two equal vectors is given by $\mathbf{A}\times\mathbf{A}=0$

  Similarly, $\hat{\mathbf{i}}\times\hat{\mathbf{i}}=(1\times1\times\sin0^\circ)\,\hat{\mathbf{n}}=0$
$$\hat{\mathbf{j}}\times\hat{\mathbf{j}}=(1\times1\times\sin0^\circ)\,\hat{\mathbf{n}}=0$$
$$\hat{\mathbf{k}}\times\hat{\mathbf{k}}=(1\times1\times\sin0^\circ)\,\hat{\mathbf{n}}=0$$
- Cross product of two perpendicular vectors is given as $\mathbf{A}\times\mathbf{B}=(AB\sin90^\circ)\,\hat{\mathbf{n}}=(AB)\,\hat{\mathbf{n}}$
- For two vectors $\mathbf{A}=a_x\hat{\mathbf{i}}+a_y\hat{\mathbf{j}}+a_z\hat{\mathbf{k}}$

  and $\mathbf{B}=b_x\hat{\mathbf{i}}+b_y\hat{\mathbf{j}}+b_z\hat{\mathbf{k}}$.
$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$
- Cross product of vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are following cyclic rules as follows $\hat{\mathbf{i}}\times\hat{\mathbf{j}}=\hat{\mathbf{k}}$, $\hat{\mathbf{j}}\times\hat{\mathbf{k}}=\hat{\mathbf{i}}$ and $\hat{\mathbf{k}}\times\hat{\mathbf{i}}=\hat{\mathbf{j}}$

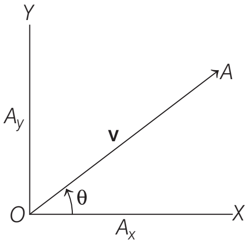


Cyclic representation for unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$

NOTE • Vector triple product is given by
$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=\mathbf{B}\,(\mathbf{A}\cdot\mathbf{C})-\mathbf{C}\,(\mathbf{A}\cdot\mathbf{B})$$

Resolution of a Vector

The process of splitting of a single vector into two or more vectors in different direction is resolution of a vectors. Consider a vector A in the X-Y plane making an angle θ with the X-axis. The X and Y components of A are A_x and A_y respectively.



Thus $\mathbf{A}_x=\mathbf{A}_{xi}=(A\cos\theta)\hat{\mathbf{i}}$ along X-direction

$$\mathbf{A}_y=\mathbf{A}_{yj}=(A\sin\theta)\hat{\mathbf{j}}\text{ along }Y\text{-direction}$$

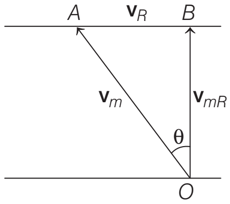
From triangle law of vector addition
$$|\mathbf{A}|=|\mathbf{A}_{xi}+\mathbf{A}_{yj}|=\sqrt{A_x^2+A_y^2}$$
and
$$\tan\theta=\frac{A_y}{A_x}=\theta=\tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Relative Velocity

The time rate of change of relative position of one object with respect to another is called relative velocity.

Different Cases

Case I If both objects A and B move along parallel straight lines in the opposite direction, then relative velocity of B w.r.t. A is given as,
$$\mathbf{v}_{BA}=\mathbf{v}_B-(-\mathbf{v}_A)=\mathbf{v}_B+\mathbf{v}_A$$
If both objects A and B move along parallel staight lines in the same direction, then
$$\mathbf{v}_{AB}=\mathbf{v}_B-\mathbf{v}_A$$
Case II Crossing the River To cross the river over shortest distance, i.e. to cross the river straight, the man should swim upstream making an angle θ with **OB** such that, **OB** gives the direction of resultant velocity $(\mathbf{v}_{mR})$ of velocity of swimmer $\mathbf{v}_M$ and velocity of river water $\mathbf{v}_R$ as shown in figure. Let us consider

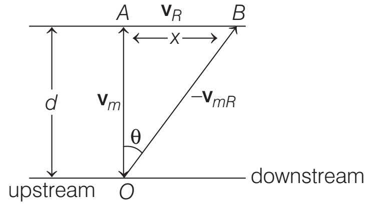


In ΔOAB, $\sin\theta=\dfrac{v_R}{v_m}$ and $v_{mR}=\sqrt{v_m^2-v_R^2}$

The time taken to cross the river given by
$$t_1=\frac{d}{v_{mR}}=\frac{d}{\sqrt{v_m^2-v_R^2}}$$
Case III To cross the river in possible shortest time The man should go along OA. Now, the swimmer will be going along OB, which is the direction of resultant velocity $\mathbf{v}_{mR}$ of v_m and v_R.

In ΔOAB, $\tan\theta=\dfrac{AB}{OA}=\dfrac{v_R}{v_m}$

and $$v_{mR}=\sqrt{v_m^2+v_R^2}$$



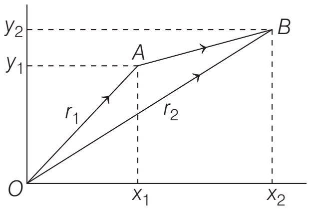
Time of crossing the river,
$$t=\frac{d}{v_m}=\frac{OB}{v_{mR}}=\frac{\sqrt{x^2+d^2}}{\sqrt{v_m^2+v_R^2}}$$
The boat will be reaching the point B instead of point A. If $AB=x$,

then, $$\tan\theta=\frac{v_R}{v_m}=\frac{x}{d}\Rightarrow x=\frac{dv_R}{v_m}$$

Motion in a Plane

Let the object be at position A and B at timing t_1 and t_2, where $OA = r_1$, and $OB = r_2$

Suppose O be the origin for measuring time and position of the object (see figure).



- Displacement of an object form position A to B is

$$AB = r = r_2 - r_1 = (x_2 - x_1)i - (y_2 - y_1)j$$

- Velocity, $v = \dfrac{r_2 - r_1}{t_2 - t_1}$
- A particle moving in X-Y plane (with uniform velocity) then, its equation of motion for X and Y axes are

$$v = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}},\ r_0 = x_0\hat{\mathbf{i}} + y_0\hat{\mathbf{j}} \text{ and } r = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$x = x_0 + v_x t,\ y = y_0 + v_y t$$

- A particle moving in xy-plane (with uniform acceleration), then its equation of motion for X and Y-axes are
$v_x = u_x + a_x t, \quad v_y = u_y + a_y t$

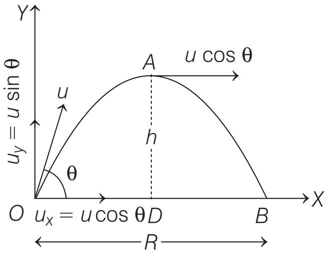
$$x = x_0 + u_x t + \frac{1}{2}a_x t^2,\ y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

$$a = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{i}}$$

Projectile Motion

Projectile is an object which once projected in a given direction with given velocity and is then free to move under gravity alone. The path described by the projectile is called its trajectory.

Let a particle is projected at an angle θ from the ground with initial velocity u.



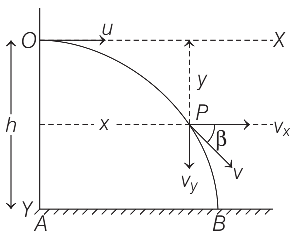
Resolving u in two components, we have

$u_x = u\cos\theta,\ u_y = u\sin\theta,\ a_x = 0,\ a_y = -g.$

- Equation of trajectory, $y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}x^2$
- Vertical height covered, $h = \dfrac{u^2\sin^2\theta}{2g}$
- Horizontal range, $R = OB = u_x\ T,\ R = \dfrac{u^2\sin 2\theta}{g}$

Projectile Motion in Horizontal Direction From Height (h)

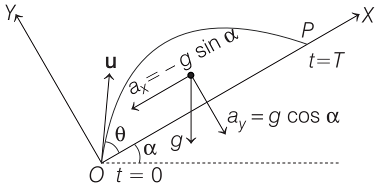
Let a particle be projected in horizontal direction with speed u from height h.



- Equation of trajectory, $y = \dfrac{gx^2}{2u^2}$
- Time of flight, $T = \dfrac{\sqrt{2h}}{g}$
- Horizontal range, $R = u\sqrt{\dfrac{2h}{g}}$
- Velocity of projectile at any time, $v = \sqrt{u^2 + g^2t^2}$

Projectile Motion Up an Inclined Plane

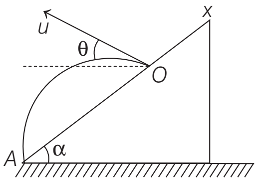
Let a particle be projected up with speed u from an inclined plane which makes an angle α with the horizontal and velocity of projection makes an angle θ with the inclined plane.

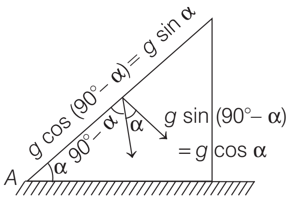


- Time of flight on an inclined plane $T = \dfrac{2u\sin\theta}{g\sin\alpha}$
- Maximum height, $h = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$
- Horizontal range, $R = \dfrac{2u^2}{g}\dfrac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$
- Maximum range occurs when $\theta = \dfrac{\pi}{2} - \dfrac{\alpha}{2}$
- $R_{\max} = \dfrac{u^2}{g(1+\sin\alpha)}$ when projectile is thrown upwards.
- $R_{\max} = \dfrac{u^2}{g(1-\sin\alpha)}$ when projectile is thrown downwards.

Projectile Motion Down an Inclined Plane

A projectile is projected down the plane from the point O with an initial velocity u at an angle θ with horizontal. The angle of inclination of plane with horizontal α. Then,





- Time of flight down an inclined plane, $T = \dfrac{2u\sin(\theta+\alpha)}{g\cos\alpha}$
- Horizontal range, $R = \dfrac{u^2}{g\cos^2\alpha}[\sin(2\theta+\alpha) + \sin\alpha]$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 Which of the following statement is true?

(a) A scalar quantity is the one that is conserved in a process

(b) A scalar quantity is one that can never be negative values

(c) A scalar quantity is the one that does not vary from one point to another in space

(d) A scalar quantity has the same value for observers with different orientations of the axes

2 If two vectors are equal in magnitude and their resultant is also equal in magnitude to one of them, then the angle between the two vectors is

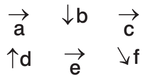
(a) 60° (b) 120° (c) 90° (d) 0°

3 If $\mathbf{A} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$ and $\mathbf{B} = 7\,\hat{\mathbf{i}} + 24\,\hat{\mathbf{j}}$, the vector having the same magnitude as **B** and parallel to **A** is

(a) $5\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}}$ (b) $15\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$ (c) $20\hat{\mathbf{i}} + 15\,\hat{\mathbf{j}}$ (d) $15\,\hat{\mathbf{i}} + 20\hat{\mathbf{j}}$

4 Six vectors **a** through **f** have the magnitudes and directions as shown in figure. Which statement is true?

➞ CBSE AIPMT 2010



(a) $\mathbf{b} + \mathbf{c} = \mathbf{f}$ (b) $\mathbf{d} + \mathbf{c} = \mathbf{f}$

(c) $\mathbf{a} + \mathbf{e} = \mathbf{f}$ (d) $\mathbf{b} + \mathbf{e} = \mathbf{f}$

5 The component of vector $\mathbf{A} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$ along the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is

(a) $\frac{5}{\sqrt{2}}$ (b) $10\sqrt{2}$ (c) $5\sqrt{2}$ (d) 5

6 **A** and **B** are two vectors and θ is the angle between them, if $|\mathbf{A} \times \mathbf{B}| = \sqrt{3}\,(\mathbf{A} \cdot \mathbf{B})$, the value of θ is

(a) 60° (b) 45° (c) 30° (d) 90°

7 Given $\mathbf{A} = 4\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. Which of the following is correct?

(a) $\mathbf{A} \times \mathbf{B} = 0$ (b) $\mathbf{A} \cdot \mathbf{B} = 24$

(c) $\frac{|\mathbf{A}|}{|\mathbf{B}|} = \frac{1}{2}$ (d) **A** and **B** are anti-parallel

8 If $\mathbf{A} = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, then angle between vectors **A** and **B** is

(a) 180° (b) 90° (c) 45° (d) 0°

9 If two vectors $2\,\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $-4\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \lambda\hat{\mathbf{k}}$ are parallel to each other, then value of λ is

(a) zero (b) −2 (c) 3 (d) 4

10 If $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \times \mathbf{B}$, then the angle between **A** and **B** is

(a) 45° (b) 30° (c) 60° (d) 90°

11 If a vector $2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + 8\,\hat{\mathbf{k}}$ is perpendicular to the vector $4\hat{\mathbf{j}} - 4\hat{\mathbf{i}} + \alpha\hat{\mathbf{k}}$, then value of α is

(a) − 1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

12 At what angle should the two forces $2P$ and $\sqrt{2}P$ act, so that the resultant force is $P\sqrt{10}$?

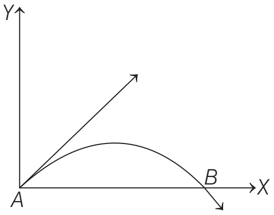
(a) 45° (b) 60° (c) 90° (d) 120°

13 A boat is sent across a river with a velocity of 8 km/h. If the resultant velocity of boat is 10 km/h, then velocity of river is

(a) 10 km/h (b) 8 km/h (c) 6 km/h (d) 4 km/h

14 The velocity of a projectile at the initial point A is $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ m/s. Its velocity (in m/s) at point B is

➞ NEET 2013



(a) $-2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ (b) $-2\mathbf{i} + 3\mathbf{j}$ (c) $2\mathbf{i} - 3\mathbf{j}$ (d) $2\mathbf{i} + 3\mathbf{j}$

15 The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in metres and t in seconds. The acceleration of the particle at $t = 2$ s is

➞ NEET 2017

(a) 0 (b) $5\hat{\mathbf{i}}$ m/s^2 (c) $-4\hat{\mathbf{i}}$ m/s^2 (d) $-8\hat{\mathbf{i}}$ m/s^2

16 A particle has initial velocity $(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ and has acceleration $(0.4\,\hat{\mathbf{i}} + 0.3\,\hat{\mathbf{j}})$. Its speed after 10 s is

➞ CBSE AIPMT 2010

(a) 7 unit (b) $7\sqrt{2}$ unit (c) 8.5 unit (d) 10 unit

17 A particle is moving such that its position coordinates (x, y) are (2 m, 3 m) at time $t = 0$, (6 m, 7 m) at time $t = 2$ s and (13 m, 14 m) at time $t = 5$ s. Average velocity vector $(\mathbf{v}_{av})$ from $t = 0$ to $t = 5$ s is

(a) $\frac{1}{5}(13\,\hat{\mathbf{i}} + 14\hat{\mathbf{j}})$ (b) $\frac{7}{3}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (c) $2\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (d) $\frac{11}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

18 The horizontal range and maximum height attained by a projectile are R and H, respectively. If a constant horizontal acceleration $a = g/4$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

(a) $(R + H), \frac{H}{2}$ (b) $\left(R + \frac{H}{2}\right), 2H$

(c) $(R + 2H), H$ (d) $(R + H), H$

19 A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 m/s. Then, the time after which its inclination with the horizontal is 45°, is

(a) 15 s (b) 10.98 s (c) 5.49 s (d) 2.745 s

20 The velocity of a particle is $v = v_0 + gt + at^3$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is

(a) $v_0 = \frac{g}{2} + a$ (b) $v_0 = 2g + 3a$

(c) $v_0 + \frac{g}{2} + \frac{a}{3}$ (d) $v_0 + g + a$

21 A projectile is fired from the surface of the earth with a velocity of 5 m/s and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 m/s at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in m/s^2) (given, $g = 9.8$ m/s^2) → CBSE AIPMT 2014

(a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8

22 The horizontal range and maximum height of a projectile are equal. The angle of projection is → CBSE AIPMT 2012

(a) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (b) $\theta = \tan^{-1}(4)$

(c) $\theta = \tan^{-1}(2)$ (d) $\theta = 45°$

23 A missile is fired for maximum range with an initial velocity of 20 m/s. If $g = 10$ m/s^2, the range of missile is → CBSE AIPMT 2011

(a) 50 m (b) 60 m

(c) 20 m (d) 40 m

24 A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of angular momentum of projectile about the point of projection when the particle is at its maximum height h is

(a) zero (b) $\frac{mvh}{\sqrt{2}}$

(c) $\frac{mvh^2}{\sqrt{2}}$ (d) None of these

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The sum of the magnitudes of two forces acting at a point is 16 N. The resultant of these forces is perpendicular to the smaller forces and has a magnitude of 8 N. If the smaller forces of magnitude x, then the value of x is

(a) 2 N (b) 4 N (c) 6 N (d) 7 N

2 If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is → NEET 2016, CBSE AIPMT 1991

(a) 90° (b) 45° (c) 180° (d) 0°

3 The value of n so that vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $5\hat{\mathbf{i}} + n\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ may be coplanar, will be

(a) 18 (b) 28 (c) 9 (d) 36

4 A projectile is given an initial velocity of $(i + 2j)$ m/s, when i is along the ground and j is along the vetical. If $g = 10$ m/s^2, the equation of its trajectory is

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

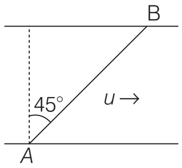
5 A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72 km/h. The jeep follows it at a speed of 90 km/h, crossing the turning 10 s later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike? (in km)

(a) 1 (b) 2 (c) 3 (d) 4

6 A boat takes 2 h to travel 8 km and back in still water. If the velocity of water 4 km/h, the time taken for going up stream 8 km and coming back is

(a) 2 h (b) 2 h 40 min (c) 1 h 20 min

(d) Cannot be estimated with the information given



7 A man wants to reach point B on the opposite bank of a river flowing at a speed as shown in figure. What minimum speed relative to water should the man have, so that he can reach point B ?

(a) $u\sqrt{2}$ (b) $u/\sqrt{2}$ (c) $2u$ (d) $u/2$

8 A particle starting from the origin (0, 0) moves in a straight line in the XY-plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the X-axis an angle of

(a) 30° (b) 45° (c) 60° (d) 0°

9 A ball is rolled off along the edge of the table with horizontal with velocity 4 m/s. It hits the ground after time 0.4 s. Which of the following statement is wrong. $(g = 10$ m/s$^2)$

(a) The height of table is 0.8 m.
(b) It hits the ground of an angle of 60° with the vertical.
(c) It covers a horizontal distance 1.6 m from the table.
(d) It hits the ground with vertical velocity 4 m/s.

10 A ship A is moving Westwards with a speed of 10 km/h and ship B 100 km South of A, is moving Northwards with a speed of 10 km/h. The time after which the distance between them becomes shortest, is ➔ CBSE AIPMT 2015

(a) 0 h (b) 5 h (c) $5\sqrt{2}$ h (d) $10\sqrt{2}$ h

11 Two particles A and B, move with constant velocities $\mathbf{v}_1$ and $\mathbf{v}_2$. At the initial moment, their position vectors are $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. The condition for particles A and B for their collision is ➔ CBSE AIPMT 2015

(a) $\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 - \mathbf{v}_1|}$ (b) $\mathbf{r}_1 \cdot \mathbf{v}_1 = \mathbf{r}_2 \cdot \mathbf{v}_2$

(c) $\mathbf{r}_1 \times \mathbf{v}_1 = \mathbf{r}_2 \times \mathbf{v}_2$ (d) $\mathbf{r}_1 - \mathbf{r}_2 = \mathbf{v}_1 - \mathbf{v}_2$

12 The position vector of a particle **R** as a function of time is given by $\mathbf{R} = 4\sin(2\pi t)\,\hat{\mathbf{i}} + 4\cos(2\pi t)\,\hat{\mathbf{j}}$

where R is in metre, t is in seconds and $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ denote unit vectors along x and y-directions, respectively. Which one of the following statements is wrong for the motion of particle? ➔ CBSE AIPMT 2015

(a) Acceleration is along $-\mathbf{R}$

(b) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle

(c) Magnitude of the velocity of particle is 8 m/s

(d) Path of the particle is a circle of radius 4 m

ANSWERS

SESSION 1	**1** (b)	**2** (b)	**3** (d)	**4** (d)	**5** (a)	**6** (a)	**7** (a)	**8** (b)	**9** (b)	**10** (a)
	11 (c)	**12** (a)	**13** (c)	**14** (c)	**15** (c)	**16** (b)	**17** (d)	**18** (d)	**19** (c)	**20** (c)
	21 (a)	**22** (b)	**23** (d)	**24** (b)						
SESSION 2	**1** (c)	**2** (a)	**3** (a)	**4** (b)	**5** (a)	**6** (b)	**7** (b)	**8** (c)	**9** (b)	**10** (b)
	11 (a)	**12** (c)								

Hints and Explanations

SESSION 1

1 A scalar quantity has same value for observers with different orientation of the axes. Since, value of scalar is independent of the direction of its observation.

2 Given, $\mathbf{R} = \mathbf{A} = \mathbf{B}$

$\therefore \quad R^2 = R^2 + R^2 + 2RR\cos\theta$

or $\cos\theta = -\frac{1}{2}$;

$\therefore \quad \theta = 120°$

3 A vector parallel to **A** will be $n\,\mathbf{A}$ or $(3n\,\hat{\mathbf{i}} + 4n\,\hat{\mathbf{j}})$

Now, $|n\mathbf{A}| = |\mathbf{B}|$ is given

Hence,

$n\sqrt{9+16} = \sqrt{49+576}$

or $\quad n = 5$

$\therefore \quad n\mathbf{A} = 15\hat{\mathbf{i}} + 20\hat{\mathbf{j}}$

4 When two non-zero vectors are represented by the two adjacent sides of a parallelogram, then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors $\mathbf{b} + \mathbf{e} = \mathbf{f}$.

5 Component of **A** along $\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$$\Rightarrow \mathbf{A}\cdot\hat{\mathbf{B}} = \mathbf{A}\cdot\frac{\mathbf{B}}{|\mathbf{B}|} = \frac{(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

6 Given, $\quad |\mathbf{A}\times\mathbf{B}| = \sqrt{3}\,(\mathbf{A}\cdot\mathbf{B})$

$\Rightarrow \quad AB\sin\theta = \sqrt{3}\,AB\cos\theta$

$\Rightarrow \quad \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

7 $\quad \mathbf{A}\times\mathbf{B} = (4\hat{\mathbf{i}} + 6\hat{\mathbf{j}})\times(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$

$= 12(\hat{\mathbf{i}}\times\hat{\mathbf{j}}) + 12(\hat{\mathbf{j}}\times\mathbf{i})$

$= 12(\hat{\mathbf{i}}\times\hat{\mathbf{j}}) - 12(\hat{\mathbf{i}}\times\hat{\mathbf{j}}) = 0$

Again, $\quad \mathbf{A}\cdot\mathbf{B} = (4\hat{\mathbf{i}} + 6\hat{\mathbf{j}})\cdot(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$

$= 8 + 18 = 26$

Again, $\quad \frac{|\mathbf{A}|}{|\mathbf{B}|} = \frac{\sqrt{16+36}}{\sqrt{4+9}} \neq \frac{1}{2}$

Also, $\quad \mathbf{B} = \frac{1}{2}\mathbf{A}$

$\Rightarrow$ **A** and **B** are parallel and not anti-parallel.

8 $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

Given, $\mathbf{A} = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}, \mathbf{B} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$\Rightarrow \quad \mathbf{A}\cdot\mathbf{B} = (4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}})\cdot(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$= 4\times 3 + 4 - 16 = 0$

$\Rightarrow \mathbf{A}\cdot\mathbf{B} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

9 The coefficients of **i**, **j**, **k** should be a constant ratio.

or $\quad \frac{2}{-4} = \frac{3}{-6} = \frac{1}{\lambda}$ or $\lambda = -2$

10 Given, $\mathbf{A}\cdot\mathbf{B} = \mathbf{A}\times\mathbf{B}$

$\Rightarrow AB\cos\theta = AB\sin\theta \Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \quad \tan\theta = 1 \Rightarrow \theta = 45°$

11 Let, $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$,

$\mathbf{b} = 4\hat{\mathbf{j}} - 4\hat{\mathbf{i}} + \alpha\hat{\mathbf{k}} = -4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}$

Given, $\mathbf{a}\perp\mathbf{b} \Rightarrow \mathbf{a}\cdot\mathbf{b} = 0$

$\Rightarrow \quad (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}})(-4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}) = 0$

$\Rightarrow -8 + 12 + 8\alpha = 0 \Rightarrow 8\alpha = -4$

$\Rightarrow \quad \alpha = -\frac{1}{2}$

12 Resultant, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Given, $\quad R = P\sqrt{10}, A = 2P, B = \sqrt{2}P$

$\therefore \quad P\sqrt{10} = \sqrt{4P^2 + 2P^2 + 4\sqrt{2}\,P^2\cos\theta}$

$\Rightarrow P\sqrt{10} = \sqrt{6P^2 + 4\sqrt{2}\,P^2\cos\theta}$

On, squaring both sides, we have

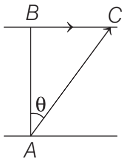
$10P^2 = 6P^2 + 4\sqrt{2}\,P^2\cos\theta$

$4P^2 = 4\sqrt{2}P^2\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

13 Given, AB = Velocity of boat = 8 km/h

AC = Resultant velocity of boat = 10 km/h



$\therefore$ BC = Velocity of river = $\sqrt{AC^2 - AB^2}$

$= \sqrt{(10)^2 - (8)^2} = 6$ km/h

14 From the figure, the x- component remain unchanged, while the y-component is reverse. Then, the velocity at point B is $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ m/s.

15 Given, $x = 5t - 2t^2$

Velocity of the particle,

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(5t - 2t^2) = 5 - 4t$$

Acceleration, $a_x = \frac{d}{dt} v_x = -4 \text{ ms}^{-2}$

Also, $y = 10t$

Velocity, $v_y = \frac{dy}{dt} = 10$

$\therefore$ Acceleration, $a_y = \frac{dv_y}{dt} = 0$

$\therefore$ Net acceleration of the particle,

$$\mathbf{a}_{\text{net}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = (-4 \text{ ms}^{-2})\,\hat{\mathbf{i}}$$

or $\mathbf{a}_{\text{net}} = -4\,\hat{\mathbf{i}} \text{ ms}^{-2}$

16 Given, initial velocity $(u) = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Final velocity $(\mathbf{v})$ = ?

Acceleration $(\mathbf{a}) = (0.4\,\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})$

Time (t) = 10 s

From first equation of motion,

$\mathbf{v} = u + at$

$\mathbf{v} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 10\,(0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})$

$\mathbf{v} = 7\hat{\mathbf{i}} + 7\hat{\mathbf{j}} \Rightarrow |\mathbf{v}| = 7\sqrt{2}$

17 Velocity, $\mathbf{v}_{\text{av}} = \frac{(x_2 - x_1)\,\hat{\mathbf{i}} + (y_2 - y_1)\,\hat{\mathbf{j}}}{t_2 - t_1}$

$$= \frac{(13-2)\,\hat{\mathbf{i}} + (14-3)\,\hat{\mathbf{j}}}{5-0}$$

$$= \frac{11\hat{\mathbf{i}} + 11\hat{\mathbf{j}}}{5} = \frac{11}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

18 $T = \frac{2u_y}{g}$, $H = \frac{u_y^2}{2g}$ and $R = u_xT$

When a horizontal acceleration is also given to the projectile u_y, T and H will remains unchanged while the range will become

$$R' = u_xT + \frac{1}{2}aT^2$$

$$= R + \frac{1}{2}\frac{g}{4}\left(\frac{4u_y^2}{g^2}\right) = R + H$$

and maximum height will be H.

19 Horizontal component of velocity at angle 60° = Horizontal component of velocity at 45°

i.e. $u \cos 60° = v \cos 45°$

or $147 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}}$ or $v = \frac{147}{\sqrt{2}}$ m/s

Vertical component of

$$u_y = u \sin 60° = \frac{147\sqrt{3}}{2} \text{ m}$$

Vertical component of

$$v_y = v \sin 45° = \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{147}{2} \text{ m}$$

but $v_y = u_y + at$

$\therefore$ $\frac{147}{2} = \frac{147\sqrt{3}}{2} - 9.8\,t$ or $t = 5.49$ s

20 Velocity $v = v_0 + gt + at^2$

$$\frac{dx}{dt} = v_0 + gt + at^2$$

Integrate on both sides,

$$\int dx = \int v_0 dt + \int gt dt + \int at^2 dt$$

$$x = v_0t + \frac{1}{2}gt^2 + \frac{at^3}{3} + C$$

Given, $x = 0$ at $t = 0$

$\therefore$ $C = 0$

$$x = v_0t + \frac{1}{2}gt^2 + \frac{1}{3}at^3$$

At t = 1 second, $x = v_0 + \frac{1}{2}g + \frac{1}{3}a$

21 $y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$

For equatorial trajectories for same angle of projection

$$\frac{8}{u^2} = \text{constant}$$

$$\Rightarrow \frac{9.8}{5^2} = \frac{g'}{3^2}$$

$$g' = \frac{9.8 \times 9}{25} = 3.528 \text{ m/s}^2$$

$$= 3.5 \text{ m/s}^2$$

22 Given, Range (R) = maximum height (H)

Also, $R = \frac{u^2\,(2\sin\theta\cos\theta)}{g}$, $H = \frac{u^2 \sin^2\theta}{2g}$

$\therefore$ $\frac{u^2\,(2\sin\theta\cos\theta)}{g} = \frac{u^2\sin^2\theta}{2g}$

$\Rightarrow 2\cos\theta = \frac{\sin\theta}{2}$

$\Rightarrow \tan\theta = 4$

$\Rightarrow \theta = \tan^{-1}(4)$

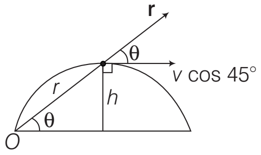
23 Maximum range of projectile is given by

$$R_{\max} = \frac{u^2}{g}$$

Given, $u = 20$ m/s and $g = 10 \text{ m/s}^2$

$\therefore$ $R_{\max} = \frac{(20)^2}{10} = \frac{400}{10} = 40$ m

24 The angular momentum of a particle is given by



$$\mathbf{L} = \mathbf{r} \times m\,\mathbf{v}$$

$\therefore$ $L = mvr \sin\theta$

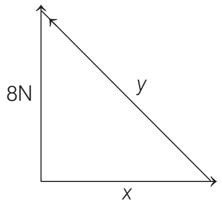
From figure,

$$L = rm\,(v \cos 45°)\sin\theta$$

$$= \frac{mv}{\sqrt{2}}(r \sin\theta) = \frac{mvh}{\sqrt{2}}$$

SESSION 2

1 Given, $x + y = 16$



Also, $y^2 = 8^2 + x^2$

or $y^2 = 64 + (16 - y)^2$ $[\because x = 16 - y]$

or $y^2 = 64 + 256 + y^2 - 32y$

or $32y = 320$ or $y = 10$ N

$\therefore$ $x + 10 = 16$ or $x = 6$ N

2 Suppose two vectors are P and Q.

It is given that $|\mathbf{P} + \mathbf{Q}| = |\mathbf{P} - \mathbf{Q}|$

Let angle between P and Q is ϕ.

$\therefore$ $P^2 + Q^2 + 2PQ \cos\phi$

$= P^2 + Q^2 - 2PQ \cos\phi$

$\Rightarrow$ $4PQ \cos\phi = 0$

$\Rightarrow$ $\cos\phi = 0$ $[\because P, Q \neq 0]$

$\Rightarrow$ $\phi = \frac{\pi}{2} = 90°$

3 For given vectors to be coplanar,

$\mathbf{A} \times \mathbf{B} \times \mathbf{C} = 0$

$\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}} \Rightarrow \mathbf{B} = 5\hat{\mathbf{i}} + n\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\therefore$ $\mathbf{C} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$$\begin{vmatrix} 2 & 3 & -2 \\ 5 & n & 1 \\ -1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2(3n - 2) - 3(15 + 1) - 2\,(10 + n) = 0$

$\Rightarrow$ $6n - 4 - 45 - 3 - 20 - 2n = 0$

$\Rightarrow$ $4n = 72$, $n = 18$

4 The equation of trajectory of a particle, fired, with an initial velocity u at an angle of projection θ,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$

$$= x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

Now, magnitude of velocity vector

$u = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} \Rightarrow u = \sqrt{(1)^2 + (2)^2} = 5\,\text{m/s}$

and angle of projection is given by

$$\tan\theta = \frac{\hat{\mathbf{j}}\text{ component}}{\hat{\mathbf{i}}\text{ component}} = \frac{2}{1} = 2$$

$$\tan\theta = 2$$

So, from eq (i), we have

$$y = 2x - \frac{10 \times x^2}{2 \times 5}(1 + 4) = 2x - 5x^2$$

5 $v_p = 90\,\text{km/h} = 25\,\text{m/s}$

$v_c = 72\,\text{km/h} = 20\,\text{m/s}$

In 10 s culprit reaches point *B* from *A*.
Distance covered by culprit,
$S = vt = 20 \times 10 = 200$ m
At time $t = 10$ s, the police jeep is 200 m behind the culprit.
Relative velocity between jeep and culprit is $25 - 20 = 5\,\text{m/s}$

$$\text{Time} = \frac{S}{v} = \frac{200}{5} = 40\text{ s}$$

[Relative velocity is considered]

In 40 s, the police jeep will move from *A* to a distance *S*

where, $S = vt = 25 \times 40 = 1000$ m
$= 1$ km away

The jeep will catch up with the bike 1 km far from the turning.

6 Boat covers distance of 16 km in a still water in 2 h

i.e. $v_B = \frac{16}{2} = 8\,\text{km/h}$

Now, velocity of water $v_W = 4$ km/h
Time taken for going upstream

$$t_1 = \frac{8}{v_B - v_W} = \frac{8}{8 - 4} = 2\text{ h}$$

As water current oppose the motion of boat, therefore time taken for going downstream

$$t_2 = \frac{8}{v_G + v_W} = \frac{8}{8 + 4} = \frac{8}{12}\text{ h}$$

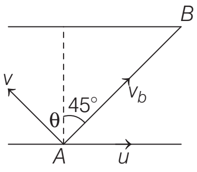
[water current helps the motion of boat]

$\therefore$ Total time $= t_1 + t_2$

$$= \left(2 + \frac{8}{12}\right)\text{h}$$

$= 2$ h 40 min

7 Let *v* be the speed of boatman in still water,



Resultant of *v* and *u* should be along *AB*. Components of v_b (absolute velocity of boatman) along *x* and *y* directions are,

$$v_x = u - v\sin\theta$$

and $v_y = v\cos\theta$

Further, $\tan 45° = \frac{v_y}{v_x}$

or $1 = \frac{v\cos\theta}{u - v\sin\theta}$

$$v = \frac{u}{\sin\theta + \cos\theta}$$

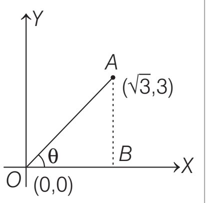
$$= \frac{u}{\sqrt{2}\sin(\theta + 45°)}$$

v is minimum at,

$$\theta + 45° = 90° \text{ or } \theta = 45°$$

and $v_{\min} = \frac{u}{\sqrt{2}}$

8 Draw the situation as shown. *OA* represents the path of the particle starting from origin *O*(0, 0). Draw a perpendicular from point *A* to *X*-axis. Let path of the particle makes an angle θ with the *X*-axis, then



$\tan\theta$ = slope of line *OA*

$$= \frac{AB}{OB} = \frac{3}{\sqrt{3}} = \sqrt{3} \text{ or } \theta = 60°$$

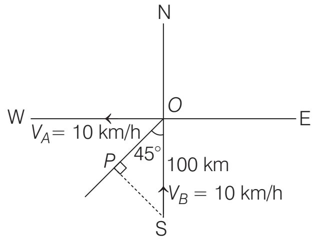
9 Height of table

$$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.4)^2 = 0.8\text{ m}$$

Horizontal distance covered $= u_x t$
$= 4 \times 0.4 = 1.6$ m
Vertical velocity on reaching ground
$v_y = u_y + a_y t = 0 + 10 \times 0.4 = 4$ m/s
Horizontal velocity on reaching ground
$v_c = u_x = 4$ m/s
If θ is the angle at which the ball hits the ground with the vertical, then

$$\tan\theta = \frac{v_x}{v_y} = \frac{4}{4} = 1 \Rightarrow \theta = 45°$$

10



$$\sin 45° = \frac{PS}{OS}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{PS}{100}$$

$$PS = \frac{100}{\sqrt{2}} = \frac{100\sqrt{2}}{2} = 50\sqrt{2}$$

Relative velocity between *A* and *B* is

$$v_{BA} = \sqrt{v_A^2 + v_B^2} = 10\sqrt{2}$$

$$t = \frac{50\sqrt{2}}{10\sqrt{2}}$$

$\Rightarrow t = 5$ h

11 For two particles *A* and *B* move with constant velocities v_1 and v_2. Such that two particles to collide, the direction of the relative velocity of one with respect to other should be directed towards the relative position of the other particle.

i.e. $\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \longrightarrow$ direction of relative position of 1 w.r.t. 2.

Similarly, $\frac{\mathbf{v}_1 - \mathbf{v}_2}{|\mathbf{v}_1 - \mathbf{v}_2|} \longrightarrow$ direction of velocity of 2 w.r.t. 1.

So, for collision of *A* and *B*, we get

$$\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 - \mathbf{v}_1|}$$

12 (i) The position vector of a particle **R** as a function of time is given by

$$\mathbf{R} = 4\sin(2\pi t)\,\hat{\mathrm{i}} + 4\cos(2\pi t)\hat{\mathrm{j}}$$

x-component,

$$x = 4\sin 2\pi t \qquad \ldots\text{(i)}$$

y-component,

$$y = 4\cos 2\pi t \qquad \ldots\text{(ii)}$$

Squaring and adding both equations, we get

$x^2 + y^2 = 4^2[\sin^2(2\pi t) + \cos^2(2\pi t)]$

i.e. $x^2 + y^2 = 4^2$ i.e. equation of circle and radius is 4 m.

(ii) Acceleration vector,

$\text{a} = \frac{v^2}{R}(-\hat{\mathbf{R}})$, while *v* is velocity of a particle.

(iii) Magnitude of acceleration vector,

$$a = \frac{v^2}{R}$$

(iv) As, we have $v_x = +\,4(\cos 2\pi t)\,2\pi$ and $v_y = -\,4(\sin 2\,\pi t)\,2\pi$

Net resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(8\pi)^2(\cos^2 2\pi\, t + \sin^2 2\,\pi\, t)}$$

$v = 8\pi \quad [\because \cos^2 2\pi t + \sin^2 2\pi t = 1]$

So, option (c) is incorrect.